CONVECTIVE HEAT TRANSFER IN CHANNELS HAVING VARIOUS CROSS SECTIONS

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A method is proposed for calculating symmetric and nonsymmetric convective heat transfer in channels having various cross sections. The turbulence model is based on a statistical approach to the determination of the turbulence characteristics. A zero-gradient type relationship is established between the kinetic energy of the pulsation motions and the Reynolds stresses.

Modern equipment carrying heavy heat loads generally employs flat, annular, or tubular channels for cooling. Owing to the large Reynolds numbers of the flow and the relatively short channels there is undeveloped turbulent flow over a significant part of a channel. Moreover, the heat-transfer condition may cause the channel walls to be at different temperatures, producing flow asymmetry, or there may be variation in the temperature of the walls along the length of the channel. Thus it is not always possible to use the familiar experimental relationships in heat-transfer calculations. A sufficiently well-founded turbulence model, permitting determination of heat and momentum transfer, must be used when we devise numerical methods for the calculations.

Here we shall consider a turbulence model based on a statistical approach to the determination of the turbulence characteristics. The equation system describing convective heat transfer for two-dimensional flow of a viscous nonisothermal fluid includes the continuity equation

$$\frac{\partial r^{\varphi}\rho U}{\partial x}+\frac{\partial r^{\varphi}\rho V}{\partial r}=0, \tag{1}$$

the momentum conservation equation

$$\rho U\frac{\partial U}{\partial x}+\rho V\frac{\partial U}{\partial r}=-\frac{\partial P}{\partial x}+\frac{1}{r^{\varphi}}\frac{\partial}{\partial r}\left(r^{\varphi}\mu\frac{\partial U}{\partial r}\right)-\frac{\partial}{\partial r}\rho\overline{u'v'}, \tag{2}$$

and the energy equation

$$\rho C_pU\frac{\partial T}{\partial x}+\rho C_pV\frac{\partial T}{\partial r}=\frac{1}{r^{\varphi}}\frac{\partial}{\partial r}\left(r^{\varphi}\lambda\frac{\partial T}{\partial r}\right)-$$
$$-\frac{\partial}{\partial r}\rho C_p\overline{v'\theta'}+U\frac{\partial P}{\partial x}+\mu\left(\frac{\partial U}{\partial r}\right)^2+\rho\bar{\varepsilon}. \tag{3}$$

An additional condition for fluid flow in a closed channel requires that the flow rate be constant:

$$\int_{-h}^{h}\frac{\partial r^{\varphi}\rho U}{\partial x}dr=0. \tag{4}$$

For a flat channel $\varphi = 0$.

The pulsation components occur in (2) and (3) because the system (1)-(3) is not closed. The concept of the turbulent viscosity

$$\mathrm{E}=\frac{-\rho\overline{u'v'}}{\mu\partial U/\partial r}, \tag{5}$$

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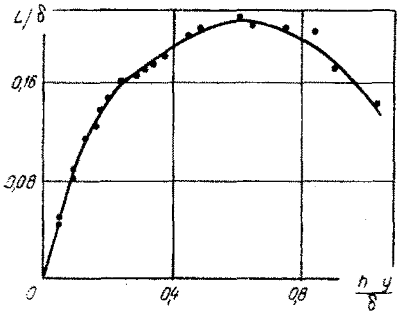

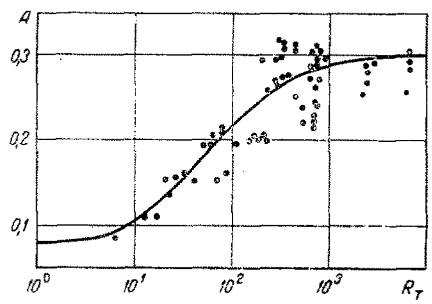

Fig. 1 Fig. 2

Fig. 1. Variation of integral turbulence scale over flow width: the dots correspond to the experimental data of Conte Bellot (channel), Favre, et al. (plate), and Zakharov, et al. (plate).

Fig. 2. Coefficient A as function of turbulence Reynolds number; the dots correspond to the experimental data of Laufer (channel), Laufer (tube), Conte Bellot (channel), Klebanov (boundary layer), Favre, et al. (boundary layer), and Bradshaw (boundary layer).

is now widely used in determination of the Reynolds stresses in (2); it is assumed to depend on either the local average flow characteristics [1, 2] or the local pulsation flow characteristics [3], or on both [4].

The analogy between the transfer of heat and momentum is ordinarily used to determine the pulsation flow of heat in (3) by introducing the turbulence Prandtl number

$$\mathrm{Pr}_T = \frac{\mu E}{-\rho\overline{v'\theta'} \Big/ \dfrac{\partial T}{\partial r}}. \tag{6}$$

As we see from the gradient representation of the Reynolds stresses (5), the extrema of the mean velocity should correspond to the zeros of these stresses. Experimental investigations [5] of nonsymmetric flows have shown, however, that the Boussinesq hypothesis is inapplicable to such flows.

Other approaches have recently been used to describe the turbulence mechanism. They make it possible to avoid the Boussinesq hypothesis and to establish direct relationships between the turbulence characteristics; we shall make use of just such an approach here. Following the ideas of [6], we write the Reynolds stresses in the form

$$-\rho\overline{u'v'} = \rho A E \gamma, \tag{7}$$

where $E = 1/2 \sum_{i=1}^{3} u_i'^2$ is the turbulence intensity which when multiplied by the density equals the kinetic energy of the pulsation motions; the intermittence coefficient γ [7] allows for the diffusion of large-scale vortices carrying Reynolds stresses of opposite sign near the channel axis. The proportionality factor A in [8] is assumed to be constant over the flow width and equal to 0.15. In [9] A is taken to equal 0.3, although doubt is expressed as to the validity of this assumption.

Here we shall take A to be a function of the turbulence Reynolds number $R_T = \sqrt{E} L\rho/\mu$, constructed from the local turbulence characteristics. Figure 1 shows the way in which the integral turbulence scale L varies over the flow width. Here y is the distance from the channel axis; δ is the distance from the wall at which the velocity u becomes equal to 0.995 times its maximum value. Figure 2 shows the form of the $A = A(R_T)$ relationship; it was obtained by processing the experimental data of various authors. The turbulence intensity E is found from the pulsation-motion kinetic-energy conservation equation which has the following form for our case [7]:

$$\rho U \frac{\partial E}{\partial x} + \rho V \frac{\partial E}{\partial r} = \frac{1}{r^{\varphi}} \frac{\partial}{\partial r}\left(r^{\varphi}\mu \frac{\partial E}{\partial r}\right) - \frac{\partial}{\partial r}\overline{v'(p' + \rho E)} - \\ -\rho\overline{u'v'}\frac{\partial U}{\partial r} - \rho\bar{\varepsilon}. \tag{8}$$

As in [8], we neglect the $\overline{v'p'}$ correlation; the pulsation diffusion of turbulence intensity is assumed to be proportional to the effective transfer rate

$$\frac{\partial}{\partial r}\overline{v'(p'-\rho E)} = \frac{\partial}{\partial r}\rho V_{ef} E, \tag{9}$$

where $V_{ef} = 0.07\sqrt{E}\gamma$.

Following [10] we write the dissipative component in the form

$$\bar{\varepsilon} = \varphi(R_T)\frac{E^{3/2}}{L}, \quad \varphi(R_T) = \frac{3,93 + 0,202R_T}{R_T}. \tag{10}$$

In determining the pulsation heat flux in (3) we decided to avoid introducing the turbulent Prandtl number and to establish direct relationships between the fields of hydrodynamic and thermal pulsating quantities, also taking into account the kind of fluid (its Prandtl number Pr). For this purpose we can write the equation for conservation of the pulsation heat flux [11]. For the case given, the solution obtained in [11] for this equation has the form

$$-\overline{v'\theta'} = \frac{\mu}{\rho}\frac{R_T^2(0,16R_T + 3,927)}{R_L^2}\cdot\frac{(0,96R_T + 3,927)}{(0,48R_T\sqrt{\mathrm{Pr}} + 3,14)}\cdot\frac{\partial T}{\partial r}, \tag{11}$$

where $R_L = (L^2\rho/\mu)(\partial U/\partial r)$ is the local Reynolds number; $\mathrm{Pr} = \mu C_p/\lambda$ is the Prandtl number.

Thus by making use of the dependences of the thermophysical properties of the fluid on the temperature and pressure we can close the system (1)-(3), (8).

The boundary conditions for the problem given take the form

$$U = V = E = 0, \; T = T_w(x). \tag{12}$$

at the wall. After solving the system (1)-(3), (8) with the appropriate initial conditions, we determine the friction stresses at the wall by means of the familiar Newton law

$$\tau_w = \mu_w\left(\frac{\partial U}{\partial r}\right)_w, \tag{13}$$

and the heat flux density at the wall from the familiar Fourier law

$$q_w = -\lambda_w\left(\frac{\partial T}{\partial r}\right)_w. \tag{14}$$

We may write (2), (3), (8) in general form as

$$a\frac{\partial f}{\partial x} + b\frac{\partial f}{\partial r} = \frac{\partial}{\partial r}\left(c\frac{\partial f}{\partial r}\right) + d + ef, \tag{15}$$

where f stands for the unknown variable for which we solve (15). The parameters a, b, c, d, e are nonlinear in the general case and may be functions of the independent variables themselves.

The flow region is partitioned by an orthogonal net; we consider the points located at the nodes of this net. To reduce the number of node points we carry out the calculations in a modified coordinate system; for a constant step it permits us to "compress" the coordinate in the r direction at the wall where the rate of change in f is greatest, and to "expand" this coordinate in the flow core where the variation in f is smoother. This corresponds to a coordinate of the form

$$z = \mathrm{tg}\,\psi r, \tag{16}$$

where $\psi = (\pi/2 - \sigma)/h$, and σ is the displacement away from $\pi/2$.

Going over to the new variable, we write the derivatives with respect to r as

$$\frac{\partial f}{\partial r} = p\frac{\partial f}{\partial z},$$

$$\frac{\partial}{\partial r}\left(c\frac{\partial f}{\partial r}\right) = \tilde{p}^2\frac{\partial}{\partial z}\left(c\frac{\partial f}{\partial z}\right) + \tilde{p}\tilde{q}c\frac{\partial f}{\partial z}, \tag{17}$$

where $\tilde{p} = \psi(1 + z^2)$; $\tilde{q} = 2z\psi$.

Substituting (17) into (15) we obtain

$$a\frac{\partial f}{\partial x} + \tilde{p}(b - c\tilde{q})\frac{\partial f}{\partial z} = \tilde{p}^2\frac{\partial}{\partial z}\left(c\frac{\partial f}{\partial z}\right) + d + ef. \tag{18}$$

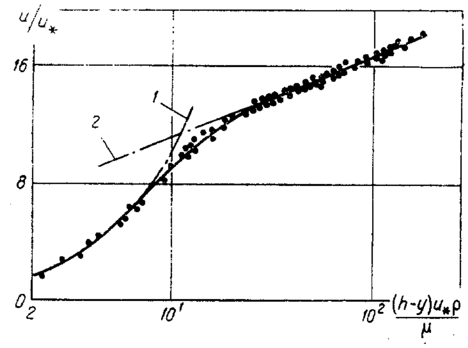

Fig. 3. Velocity profile over stabilized section of turbulent flow; solid curve) calculated; dots) experimental data of Nikuradze, Motzfeld, Shu, Reychard, Conte Bellot: 1) $u/u_* = (h-y)u_*\rho/\mu$; 2) $u/u_* = 5.5\log[(h-y)u_*\rho/\mu] + 5.5$.

We replace the differential operators in (18) by finite-difference analogs, using the implicit scheme

$$\left(\frac{\partial f}{\partial x}\right)_n^m = \frac{1}{\Delta x}(f_n^m - f_{n-1}^m),$$

$$\left(\frac{\partial f}{\partial z}\right)_n^m = \frac{1}{2\Delta z}(f_n^{m+1} - f_n^{m-1}),$$

$$\left[\frac{\partial}{\partial z}\left(c\frac{\partial f}{\partial z}\right)\right]_n^m = \frac{1}{\Delta z^2}\left[\frac{c_n^{m+1}+c_n^m}{2}(f_n^{m+1} - f_n^m) - \frac{c_n^m + c_n^{m-1}}{2}(f_n^m - f_n^{m-1})\right]. \tag{19}$$

After certain manipulations we have

$$\alpha_m f_n^{m-1} + \beta_m f_n^m + \gamma_m f_n^{m+1} = \delta_m, \tag{20}$$

where

$$\alpha_m = -\frac{1}{2\Delta z}\left[\tilde{p}_n^m(b_n^m - c_n^m q_n^m) - (\tilde{p}_n^m)^2\frac{c_n^m + c_n^{m-1}}{\Delta z}\right],$$

$$\beta_m = \frac{a_n^m}{\Delta x} - e_n^m + \frac{(\tilde{p}_n^m)^2}{2\Delta z^2}(c_n^{m+1} + 2c_n^m + c_n^{m-1}),$$

$$\gamma_m = \frac{1}{2\Delta z}\left[\tilde{p}_n^m(b_n^m - c_n^m\tilde{q}_n^m) - (\tilde{p}_n^m)^2\frac{c_n^{m+1} + c_n^m}{\Delta z}\right],$$

$$\delta_m = d_n^m + \frac{a_n^m}{\Delta x}f_{n-1}^m. \tag{21}$$

When (2) is expressed in the form (20) the right side looks like this:

$$\delta_m^* = \delta_m - \frac{\partial P}{\partial x}. \tag{22}$$

In writing (21), (22) we assumed that the z-coordinate step is constant. The continuity equation (1) is so solved in explicit form and in natural coordinates:

for a flat channel:

$$(\rho V)_n^m = (\rho V)_n^{m-1} + \frac{2(r_n^{m-1} - r_n^m)}{4\Delta x}[(\rho U)_n^m - (\rho U)_{n-1}^m + (\rho U)_n^{m-1} - (\rho U)_{n-1}^{m-1}], \tag{23}$$

while for an annular channel and a round tube

$$(\rho Vr)_n^m = (\rho Vr)_n^{m-1} + \frac{(r_n^{m-1})^2 - (r_n^m)^2}{4\Delta x}[(\rho U)_n^m - (\rho U)_{n-1}^m + (\rho U)_n^{m-1} - (\rho U)_{n-1}^{m-1}]. \tag{24}$$

Equations (20) are solved by the factorization method [12].

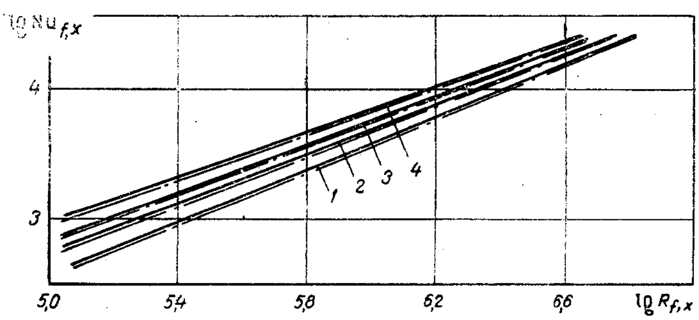

Fig. 4. Comparison of calculated and experimental data on heat transfer; solid line) calculated; dash-dot line) $Nu_{f,x} = 0.021R_{f,x}^{0.8}Pr_{f,x}^{0.43}(x/d_{equiv})^{0.2}(Pr_{f,x}/Pr_w)^{0.25}$; 1) $T_0/T_w \simeq 1.0$, 2) 0.9, 3) 0.8, 4) 0.75. $Pr_w = 1.8$.

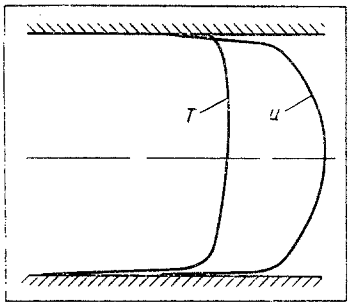

Fig. 5

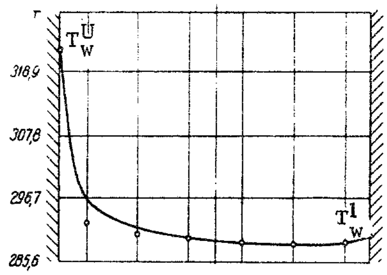

Fig. 6

Fig. 5. Velocity and temperature profiles for nonsymmetric flow in flat channel.

Fig. 6. Comparison of calculations with experimental data of [14]; solid line) calculated; dots) experimental data; T, °K.

We use the condition (4) to find the unknown pressure gradient $\partial P/\partial x$ in (2). For this purpose we represent the function $f_n^m = U_n^m$ as

$$f_n^m = U_n^m = W_n^m + K_n^m \frac{\partial P}{\partial x}. \tag{25}$$

After substituting (25) into (20) we obtain two equations for W_n^m and K_n^m:

$$\alpha_m W_n^{m-1} + \beta_m W_n^m + \gamma_m W_n^{m+1} = D_n^m, \tag{26}$$

$$\alpha_m K_n^{m-1} + \beta_m K_n^m + \gamma_m K_n^{m+1} = \Phi_n^m, \tag{27}$$

which are solvable by factorization with zero boundary conditions.

Determining W_n^m and K_n^m, we substitute (25) into (4). Then we have

$$\frac{\partial P}{\partial x} = \frac{\sum_{m=1}^{M} \nu_n^m (U_{n-1}^m - W_n^m)}{\sum_{m=1}^{M} \nu_n^m K_n^m}, \tag{A}$$

where $\nu_n^m = \rho_n^m (r_n^{m-1} - r_n^m)$ for a flat channel and $\nu_n^m = \rho_n^m [(r_n^{m-1})^2 - (r_n^m)^2]$ for an annular channel of round tube. After finding $\partial P/\partial x$ from (25) we determine the values of U_n^m. Our program was run on a BÉSM-4 computer.

We used experimental data and exact solutions obtained for simple cases of flow to check the calculation method. Laminar and turbulent flows of liquids and gases were considered. Figure 3 shows the calculated velocity profile over a stabilized section of an isothermal turbulent flow in a round tube and a flat channel. The calculations clearly agree with the experimental data of various authors. Figure 4 shows calculated data for convective heat transfer in annular and flat channels for turbulent flow of a liquid; the results are in fairly good agreement with experimental data processed as well known criterial relationships [13]. Figure 5 shows data calculated for nonsymmetric flow of a liquid in a flat channel. Figure 6 compares calculated values with the experimental data of [14] for nonsymmetric heat transfer of air in a flat channel. Our calculations have shown that it is possible to use the method developed in calculations for both simple and complex cases of heat transfer in channels having various cross sections.

NOTATION

x, r	are the rectangular coordinates along the flow and normal to the wall;
U, V	are the velocities along the x and r coordinates, respectively;
P	is the pressure;
T	is the temperature;
p'	is the pressure fluctuation;
u', v'	are the velocity pulsations along the x and r axes;
ρ	is the density;

μ	is the dynamic viscosity coefficient;
Cp	is the specific heat at constant pressure;
λ	is the thermal-conductivity coefficient;
θ'	is the temperature fluctuation;
$\bar{\varepsilon}$	is the rate of dissipation of the kinetic energy of pulsation motions;
u_*	is the dynamic velocity;
T_w^l	is the temperature of the lower channel wall;
T_w^u	is the temperature of the upper channel wall;
h	is the channel half-width.

Subscripts

o	conditions at channel inlet;
w	conditions at wall;
f	parameters determined for average-mass temperature of fluid at given channel cross section;
x	running value.

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